Calculus Name____________________________________ ID: 1

Assignment

Date________________ Period____

For each problem, find all points of absolute minima and maxima on the given interval.

1) $f(x) = -x^4 + 4x^2 + 1; \quad (-1, \infty)$

2) $y = x^3 - x^2 + 4; \quad (-\infty, -1)$

3) $f(x) = x^3 - 4x^2 + 7; \quad (1, 4]$

4) $y = -\frac{1}{5}(x - 4)^{\frac{5}{3}} - 2(x - 4)^{\frac{2}{3}} - 2; \quad (2, 5]$

5) $f(x) = \cot(x); \quad (-\frac{\pi}{2}, \frac{\pi}{6})$

6) $y = -x^3 + 3x^2 - 4; \quad (0, \infty)$

7) $f(x) = -2\csc(x); \quad [-\frac{\pi}{4}, \frac{\pi}{2}]$

8) $y = 2\sin(x); \quad (-\frac{\pi}{4}, -\frac{\pi}{6}]$

9) $f(x) = -\frac{1}{6}(x + 2)^{\frac{7}{3}} + \frac{14}{3}(x + 2)^{\frac{1}{3}} + 1; \quad (-6, -1)$

10) $y = -(x + 2)^{\frac{2}{3}}$; $(-4, -1)$

For each problem, determine if the Mean Value Theorem can be applied. If it can, find all values of *c* that satisfy the theorem. If it cannot, explain why not.

11) $f = \frac{-r^2 + 1}{2r}$; $[-6, -1]$

12) $f(t) = -\frac{t^2}{3t - 6}$; $[-2, 1]$

13) $h(w) = -(w + 3)^{\frac{2}{3}}$; $[-4, 0]$

14) $f(w) = -\frac{w^2}{3w - 6}$; $[1, 5]$

15) $f(w) = -\frac{w^2}{3w - 3}$; $[0, 3]$

Solve each optimization problem.

16) Which point on the graph of $y = \sqrt{x}$ is closest to the point $(4, 0)$?

17) Which points on the graph of $y = 4 - x^2$ are closest to the point $(0, 3)$?

18) Which point on the graph of $y = \sqrt{x}$ is closest to the point $(3, 0)$?

19) A rancher wants to construct two identical rectangular corrals using 200 ft of fencing. The rancher decides to build them adjacent to each other, so they share fencing on one side. What dimensions should the rancher use to construct each corral so that together, they will enclose the largest possible area?

20) Which points on the graph of $y = 4 - x^2$ are closest to the point $(0, 2)$?

21) Two vertical poles, one 5 ft high and the other 15 ft high, stand 48 feet apart on a flat field. A worker wants to support both poles by running rope from the ground to the top of each post. If the worker wants to stake both ropes in the ground at the same point, where should the stake be placed to use the least amount of rope?

22) Two vertical poles, one 15 ft high and the other 30 ft high, stand 24 feet apart on a flat field. A worker wants to support both poles by running rope from the ground to the top of each post. If the worker wants to stake both ropes in the ground at the same point, where should the stake be placed to use the least amount of rope?

23) A farmer wants to construct a rectangular pigpen using 200 ft of fencing. The pen will be built next to an existing stone wall, so only three sides of fencing need to be constructed to enclose the pen. What dimensions should the farmer use to construct the pen with the largest possible area?

24) Which point on the graph of $y = \sqrt{x}$ is closest to the point $(7, 0)$?

25) Which points on the graph of $y = 5 - x^2$ are closest to the point $(0, 3)$?

Use differentials to solve each problem.

26) The sides of a square are measured to be 5 ft, with a possible error of $\pm\frac{3}{10}$ ft. Estimate the possible propagated error in the calculated area.

27) The hypotenuse of a right triangle is known to be exactly 10 ft. One of the acute angles is measured to be 601°, with a possible error of ±21°. Estimate the possible propagated error in the side opposite to the measured angle.

28) The hypotenuse of a right triangle is known to be exactly 8 cm. One of the acute angles is measured to be 301°, with a possible error of ±21°. Estimate the possible propagated error in the side adjacent to the measured angle.

29) The radius of a sphere is measured to be 4 cm, with a possible error of $\pm\frac{1}{5}$ cm. Estimate the possible propagated error in the calculated surface area.

30) The hypotenuse of a right triangle is known to be exactly 10 cm. One of the acute angles is measured to be 301°, with a possible error of ±21°. Estimate the possible propagated error in the side opposite to the measured angle.

Solve each related rate problem.

31) A spherical snowball is rolled in fresh snow, causing it to grow so that its radius (r) increases at a rate of $\frac{3}{r}$ in/sec. How fast is the volume of the snowball increasing when the radius is 4 in?

32) A 17 ft ladder is leaning against a wall and sliding towards the floor. The top of the ladder is sliding down the wall at a rate of 2 ft/sec. How fast is the base of the ladder sliding away from the wall when the base of the ladder is 8 ft from the wall?

33) A spherical snowball is rolled in fresh snow, causing it grow at a rate of 36π in^3/sec. How fast is the radius of the snowball increasing when the radius is 9 in?

34) A 6 ft tall person is walking away from a 18 ft tall lamppost at a rate of 4 ft/sec. Assume the scenario can be modeled with right triangles. At what rate is the length of the person's shadow changing when the person is 18 ft from the lamppost?

35) A conical paper cup is 10 cm tall with a radius of 10 cm. The bottom of the cup is punctured so that the water level goes down at a rate of 2 cm/sec. At what rate is the volume of water in the cup changing when the water level is 6 cm?

36) A hypothetical square grows at a rate of $\frac{8}{A}$ m^2/min, where A is the area of the square. How fast are the diagonals of the square increasing when the diagonals are 4 m each?

37) A spherical balloon is inflated so that its radius (r) increases at a rate of $\frac{3}{r}$ cm/sec. How fast is the volume of the balloon increasing when the radius is 2 cm?

38) A hypothetical cube grows so that the length of its sides (s) are increasing at a rate of $\frac{2}{s}$ m/min. How fast is the volume of the cube increasing when the sides are 3 m each?

39) A conical paper cup is 10 cm tall with a radius of 10 cm. The bottom of the cup is punctured so that the water leaks out at a rate of $\frac{16\pi}{3V}$ cm^3/sec, where V is the volume of water in the cup. At what rate is the water level changing when the water level is 3 cm?

40) A crowd gathers around a movie star, forming a circle. The area taken up by the crowd increases at a rate of 16π ft^2/sec. How fast is the radius of the crowd increasing when the radius is 7 ft?

Answers to Assignment (ID: 1)

1) No absolute minima.
Absolute maximum: $\left(\sqrt{2}, 5\right)$

2) No absolute minima.
No absolute maxima.

3) Absolute minimum: $\left(\frac{8}{3}, -\frac{67}{27}\right)$
Absolute maximum: $(4, 7)$

4) No absolute minima.
Absolute maximum: $(4, -2)$

5) No absolute minima.
No absolute maxima.

6) No absolute minima.
Absolute maximum: $(2, 0)$

7) No absolute minima.
No absolute maxima.

8) No absolute minima.
Absolute maximum: $\left(-\frac{\pi}{6}, -1\right)$

9) Absolute minimum: $\left(-4, -4\sqrt[3]{2} + 1\right)$
No absolute maxima.

10) No absolute minima.
Absolute maximum: $(-2, 0)$

11) $\left\{-\sqrt{6}\right\}$

12) $\{0\}$

13) The function is not differentiable on $(-4, 0)$

14) The function is not continuous on $[1, 5]$

15) The function is not continuous on $[0, 3]$

16) d = the distance from point $(4, 0)$ to a point on the curve x = the x-coordinate of a point on the curve
Function to minimize: $d = \sqrt{(x-4)^2 + \left(\sqrt{x}\right)^2}$ where $-\infty < x < \infty$
Point on the curve that is closest to the point $(4, 0)$: $\left(\frac{7}{2}, \frac{\sqrt{14}}{2}\right)$

17) d = the distance from point $(0, 3)$ to a point on the parabola x = the x-coord. of a point on the parabola
Function to minimize: $d = \sqrt{x^2 + \left(4 - x^2 - 3\right)^2}$ where $-\infty < x < \infty$
Points on the parabola that are closest to the point $(0, 3)$: $\left(-\frac{\sqrt{2}}{2}, \frac{7}{2}\right), \left(\frac{\sqrt{2}}{2}, \frac{7}{2}\right)$

18) d = the distance from point $(3, 0)$ to a point on the curve x = the x-coordinate of a point on the curve
Function to minimize: $d = \sqrt{(x-3)^2 + \left(\sqrt{x}\right)^2}$ where $-\infty < x < \infty$
Point on the curve that is closest to the point $(3, 0)$: $\left(\frac{5}{2}, \frac{\sqrt{10}}{2}\right)$

19) A = the total area of the two corrals x = the length of the non-adjacent sides of each corral
Function to maximize: $A = 2x \cdot \frac{200 - 4x}{3}$ where $0 < x < 50$
Dimensions of each corall: 25 ft (non-adjacent sides) by $\frac{100}{3}$ ft (adjacent sides)

20) d = the distance from point $(0, 2)$ to a point on the parabola x = the x-coord. of a point on the parabola
Function to minimize: $d = \sqrt{x^2 + \left(4 - x^2 - 2\right)^2}$ where $-\infty < x < \infty$
Points on the parabola that are closest to the point $(0, 2)$: $\left(-\frac{\sqrt{6}}{2}, \frac{5}{2}\right), \left(\frac{\sqrt{6}}{2}, \frac{5}{2}\right)$

21) L = the total length of rope x = the horizontal distance from the short pole to the stake
Function to minimize: $L = \sqrt{x^2 + 5^2} + \sqrt{(48 - x)^2 + 15^2}$ where $0 \le x \le 48$
Stake should be placed: 12 ft from the short pole (or 36 ft from the long pole)

22) L = the total length of rope x = the horizontal distance from the short pole to the stake
Function to minimize: $L = \sqrt{x^2 + 15^2} + \sqrt{(24 - x)^2 + 30^2}$ where $0 \le x \le 24$
Stake should be placed: 8 ft from the short pole (or 16 ft from the long pole)

23) A = the area of the pigpen x = the length of the sides perpendicular to the stone wall
Function to maximize: $A = x(200 - 2x)$ where $0 < x < 100$
Dimensions of the pigpen: 50 ft (perpendicular to wall) by 100 ft (parallel to wall)

24) d = the distance from point $(7, 0)$ to a point on the curve x = the x-coordinate of a point on the curve

Function to minimize: $d = \sqrt{(x-7)^2 + (\sqrt{x})^2}$ where $-\infty < x < \infty$

Point on the curve that is closest to the point $(7, 0)$: $\left(\frac{13}{2}, \frac{\sqrt{26}}{2}\right)$

25) d = the distance from point $(0, 3)$ to a point on the parabola x = the x-coord. of a point on the parabola

Function to minimize: $d = \sqrt{x^2 + (5 - x^2 - 3)^2}$ where $-\infty < x < \infty$

Points on the parabola that are closest to the point $(0, 3)$: $\left(-\frac{\sqrt{6}}{2}, \frac{7}{2}\right), \left(\frac{\sqrt{6}}{2}, \frac{7}{2}\right)$

26) $A = s^2$, $dA = 2s\, ds$

$s = 5$, $ds = \pm 0.3$

$\Delta A \approx dA = \pm 3$ ft²

27) $x = 10\sin\theta$, $dx = 10\cos\theta\, d\theta$

$\theta = \frac{\pi}{3}$ radians, $d\theta = \pm\frac{\pi}{90}$ radians

$\Delta x \approx dx = \pm\frac{\pi}{18} \approx \pm 0.1745$ ft

28) $x = 8\cos\theta$, $dx = -8\sin\theta\, d\theta$

$\theta = \frac{\pi}{6}$ radians, $d\theta = \pm\frac{\pi}{90}$ radians

$\Delta x \approx dx = \pm\frac{2\pi}{45} \approx \pm 0.1396$ cm

29) $A = 4\pi r^2$, $dA = 8\pi r\, dr$

$r = 4$, $dr = \pm 0.2$

$\Delta A \approx dA = \pm\frac{32\pi}{5} \approx \pm 20.1062$ cm²

30) $x = 10\sin\theta$, $dx = 10\cos\theta\, d\theta$

$\theta = \frac{\pi}{6}$ radians, $d\theta = \pm\frac{\pi}{90}$ radians

$\Delta x \approx dx = \pm\frac{\pi\sqrt{3}}{18} \approx \pm 0.3023$ cm

31) V = volume of sphere r = radius t = time

Equation: $V = \frac{4}{3}\pi r^3$ Given rate: $\frac{dr}{dt} = \frac{3}{r}$ Find: $\left.\frac{dV}{dt}\right|_{r=4}$

$$\left.\frac{dV}{dt}\right|_{r=4} = 4\pi r^2 \cdot \frac{dr}{dt} = 48\pi \text{ in}^3\text{/sec}$$

32) x = horizontal distance from base of ladder to wall y = vertical distance from top of ladder to floor t = time

Equation: $x^2 + y^2 = 17^2$ Given rate: $\frac{dy}{dt} = -2$ Find: $\left.\frac{dx}{dt}\right|_{x=8}$

$$\left.\frac{dx}{dt}\right|_{x=8} = -\frac{y}{x} \cdot \frac{dy}{dt} = \frac{15}{4} \text{ ft/sec}$$

33) V = volume of sphere r = radius t = time

Equation: $V = \frac{4}{3}\pi r^3$ Given rate: $\frac{dV}{dt} = 36\pi$ Find: $\left.\frac{dr}{dt}\right|_{r=9}$

$$\left.\frac{dr}{dt}\right|_{r=9} = \frac{1}{4\pi r^2} \cdot \frac{dV}{dt} = \frac{1}{9} \text{ in/sec}$$

34) x = distance from person to lamppost y = length of shadow t = time

Equation: $\frac{x+y}{18}=\frac{y}{6}$ Given rate: $\frac{dx}{dt}=4$ Find: $\left.\frac{dy}{dt}\right|_{x=18}$

$$\left.\frac{dy}{dt}\right|_{x=18}=\frac{1}{2}\cdot\frac{dx}{dt}=2\text{ ft/sec}$$

35) V = volume of material in cone h = height t = time

Equation: $V=\frac{\pi h^3}{3}$ Given rate: $\frac{dh}{dt}=-2$ Find: $\left.\frac{dV}{dt}\right|_{h=6}$

$$\left.\frac{dV}{dt}\right|_{h=6}=\pi h^2\cdot\frac{dh}{dt}=-72\pi\text{ cm}^3\text{/sec}$$

36) A = area of square x = length of diagonals t = time

Equation: $A=\frac{x^2}{2}$ Given rate: $\frac{dA}{dt}=\frac{8}{A}$ Find: $\left.\frac{dx}{dt}\right|_{x=4}$

$$\left.\frac{dx}{dt}\right|_{x=4}=\frac{1}{x}\cdot\frac{dA}{dt}=\frac{1}{4}\text{ m/min}$$

37) V = volume of sphere r = radius t = time

Equation: $V=\frac{4}{3}\pi r^3$ Given rate: $\frac{dr}{dt}=\frac{3}{r}$ Find: $\left.\frac{dV}{dt}\right|_{r=2}$

$$\left.\frac{dV}{dt}\right|_{r=2}=4\pi r^2\cdot\frac{dr}{dt}=24\pi\text{ cm}^3\text{/sec}$$

38) V = volume of cube s = length of sides t = time

Equation: $V=s^3$ Given rate: $\frac{ds}{dt}=\frac{2}{s}$ Find: $\left.\frac{dV}{dt}\right|_{s=3}$

$$\left.\frac{dV}{dt}\right|_{s=3}=3s^2\cdot\frac{ds}{dt}=18\text{ m}^3\text{/min}$$

39) V = volume of material in cone h = height t = time

Equation: $V=\frac{\pi h^3}{3}$ Given rate: $\frac{dV}{dt}=-\frac{16\pi}{3V}$ Find: $\left.\frac{dh}{dt}\right|_{h=3}$

$$\left.\frac{dh}{dt}\right|_{h=3}=\frac{1}{\pi h^2}\cdot\frac{dV}{dt}=-\frac{16}{243\pi}\text{ cm/sec}$$

40) A = area of circle r = radius t = time

Equation: $A=\pi r^2$ Given rate: $\frac{dA}{dt}=16\pi$ Find: $\left.\frac{dr}{dt}\right|_{r=7}$

$$\left.\frac{dr}{dt}\right|_{r=7}=\frac{1}{2\pi r}\cdot\frac{dA}{dt}=\frac{8}{7}\text{ ft/s}$$